374

Elements of Statistics (Math 106) – Final Exam
Fall 2005 – Brad Hartlaub

Name Brad Hartlaub - Solutions

1. Role-playing games like Dungeons & Dragons use many different types of dice. Suppose that a four-sided die has faces marked with 1, 2, 3, and 4 spots. The intelligence of a character is determined by rolling this die twice and adding 1 to the sum of the spots.

a. What is the sample space for rolling the die twice (the sum of the spots on the first and second rolls)? (5)

$S = \{(1,1), (1,2), (1,3) (1,4),$
$(2,1) (2,2), (2,3) (2,4),$
$(3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$

b. What is the sample space for the character's intelligence? (5)

$S_2 = \{3, 4, 5, 6, 7, 8, 9\}$

c. Calculate the probabilities for the possible values of a character's intelligence. (10)

X	3	4	5	6	7	8	9
prob	1/16	2/16	3/16	4/16	3/16	2/16	1/16

d. What is the expected value (mean) of a character's intelligence? (5)

$$\mu_X = 3(1/16) + 4(2/16) + 5(3/16) + 6(4/16) + 7(3/16) + 8(2/16) + 9(1/16)$$
$$= 6$$

e. What is the standard deviation of a character's intelligence? (5)

$$\sigma_X^2 = (3-6)^2(1/16) + (4-6)^2(2/16) + (5-6)^2(3/16) + (6-6)^2(4/16) + (7-6)^2(3/16) + (8-6)^2(2/16) + (9-6)^2 1/16$$
$$\sigma_X^2 = 2.5 \Rightarrow \sigma_X = \sqrt{2.5} = 1.58114$$

2. A home alarm system has detectors covering n zones of the house. Suppose the probability is 0.7 that a detector sounds an alarm when an intruder passes through its zone, and this probability is the same each detector. The alarms operate independently. An intruder enters the house and passes through all the zones.

Let X = # of alarms that sound ; $X \sim$ Binomial$(n, p = 7/10)$

a. What is the probability that an alarm sounds if $n = 3$? (5)

$n=3$ $\quad P(X \geq 1) = 1 - P(X=0) = 1 - (.3)^3 = 1 - .027 = .973$

b. What is the probability that an alarm sounds if $n = 6$? Does the probability in part (a) get doubled? (8)

$n=6$ $\quad P(X \geq 1) = 1 - P(X=0) = 1 - (.3)^6 = 1 - .0007 = .9993$

c. How large must n be in order to have the probability of an alarm sounding be about 0.99? (10)

Use trial & error to find $n=4$ OR $\rightarrow 1 - (.3)^n = .99 \Leftrightarrow (.3)^n = .01$
$\Leftrightarrow n \ln(.3) = \ln(.01) \Leftrightarrow n = \frac{\ln(.01)}{\ln(.3)} = 3.82$ $\boxed{n=4}$

d. What is the probability that exactly five alarms go off if $n = 6$? (5)

$P(X=5) = .302526$

e. What is the probability that at most four alarms go off if $n = 6$? (5)

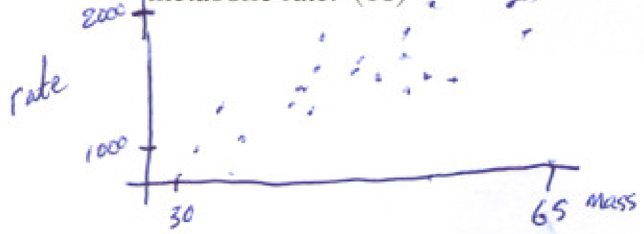
$P(X \leq 4) = .579825$

f. What is the probability that less than four alarms go off if $n = 6$? (5)

$P(X < 4) = P(X \leq 3) = .25569$

3. Metabolic rate, the rate at which the body consumes energy, is important in studies of weight gain, dieting, and exercise. The file p:\data\math\stats\metabolic.mtw contains data on the lean body mass and resting metabolic rate for 12 women and 7 men who are subjects in a study of dieting. Lean body mass, given in kilograms, is a person's weight leaving out all fat. Metabolic rate is measured in calories burned per 24 hours, the same calories used to describe energy content in foods. The researchers believe that lean body mass is an important influence on metabolic rate.

a. Is there a relationship between lean body mass and metabolic rate for these individuals? Sketch an appropriate plot, explain what the plot illustrates, and provide a descriptive statistic that measures the strength of linear association between lean body mass and metabolic rate. (10)



The plot clearly shows a strong positive association between metabolic rate and lean body mass. As mass increases, metabolic rate also tends to increase.

$r = .865$ $\quad$ R-sq = 74.8%

b. Find the least squares line for predicting metabolic rate from lean body mass and interpret the estimates of the slope and intercept parameters in the context of this question. (10)

$\widehat{rate} = 113.2 + 26.879\ mass$

slope: for every 1 unit increase in mass, the metabolic rate increases by 26.879 units.

intercept has no practical meaning in this situation - metabolic rate when there is no mass??

c. Is there a significant linear association between metabolic rate and lean body mass? Conduct an appropriate hypothesis test using a significance level of 0.05. Provide the hypotheses, test statistic, p-value, and conclusion. (15)

$H_0: \beta = 0$ $\quad t = 7.10$

$H_1: \beta \neq 0$ $\quad$ p-value = 0.000

Since 0.000 < .05, we reject H_0 and conclude that there is a significant linear relationship between metabolic rate & lean body mass.

d. Run the regression to predict metabolic rate from lean body mass for the women in the sample and summarize your results. How does this model compare with the model for all individuals in parts (a) – (c)? (15)

$\widehat{rate}_f = 201.2 + 24.026\ mass_f$

The two models are very similar. The slope for the women is a little smaller (24.026 vs 26.879) and the intercept is larger (201.2 vs 113.2). The percentage of variation in rate that is explained by mass is 74.8% for women and 76.8% for all individuals

e. What percentage of the variation in metabolic rates for the women is explained by using linear regression with the explanatory variable lean body mass? (5)

R-sq = 76.8%

With Rounded MTB output

f. One of the women has a lean body mass of 34.5 kg, find the predicted value and the residual for this woman. (6)

predicted = 1030.06 = 201.2 + 24.026(34.5)

resid = 1052 − 1030.06 = 21.94

1029

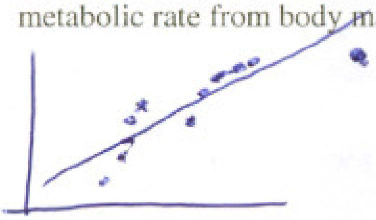
23

g. Would you be willing to use your regression model for women to predict the metabolic rate for a woman with a lean body mass of 40 kg? If so, provide the appropriate predicted value and a 95% prediction interval. If not, explain why not. (10)

Yes, there is one large residual, but overall the model provides a good fit lean body masses between 34 and 55. Predicted value = 1162.2 (1161)

95% Prediction interval: (939.9, 1384.5)

h. Construct a normal probability plot of the residuals for the women and explain what this graph tells you about the assumption of normality for the error terms when predicting metabolic rate from body mass. (5)



There is one unusually large residual and the normal probability plot shows some clear departures from linear trend. Thus, the error terms do not appear to be normally distributed.

i. Is there a significant difference in the mean metabolic rates for men and women? State the appropriate hypotheses, compute the test statistic and p-value, and provide an appropriate conclusion in the context of this problem. (15)

$H_0: \mu_f = \mu_m$ vs $H_1: \mu_f \neq \mu_m$

Probability plots indicate that the rates for men & women are approx normal (both show linear trend)

Test stat: $t = -4.06$ d.f. = 12

p-value = .002

Conclusion: Since .002 < .05, we reject H_0 and conclude that the mean metabolic rates are significantly different for men & women.

j. Would the means and standard deviations of the lean body masses for the men and women change if the measurements were transformed from kilograms to pounds? You do not need to provide any calculations. Just provide an explanation for your response. (10)

Yes, the means and standard deviations would change because changing the scale with a linear transformation affects both measures of center and measures of spread.

4. An undesired side effect of some antihistamines is drowsiness, which can be measured by the flicker frequency of patients (number of flicks of the eyelids per minute). Low flicker frequency is related to drowsiness because the eyes are staying shut too long. One study reported data for nine patients (see p:\data\math\stats\flicker.mtw), each subjected to meclastine (A), a placebo (B), and promethazine (C), in random order. At the time of the study, A was a new drug and C was a standard drug known to cause drowsiness.

Lower flicker rates for A.

a. Is there sufficient evidence to say that drug A causes more drowsiness ~~that~~ than drug B? Be sure to include all parts of the appropriate inference procedure. (15)

Paired data => form differences A-B for each patient

$H_0: \mu_d = 0$ vs $H_1\ \mu_d < 0$ Normal prob plot is roughly linear.

$t = -1.51$, p-value $= .085$

Since $.085 > .05$, we do not have significant evidence that drug A causes more drowsiness.

b. Estimate the difference between treatments B and C. Identify the parameter(s) of interest and provide a point and interval estimate. (10)

Again, data are paired (3 obs on each patient) so form differences C-B for each patient. The mean difference (C-B) for the 9 patients is -1.6133 and a 95% CI for the mean difference (C-B) is $(-2.5929, -.63376)$. Notice that zero is not in the CI for μ_d = the mean difference in the flicker rates for treatment C - placebo.

c. What treatment would you recommend? Provide an explanation for your choice. (15)

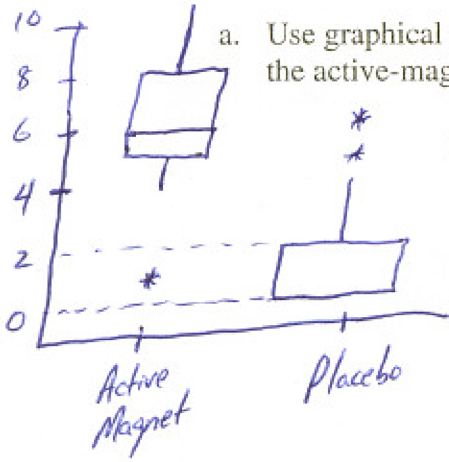
If the recommendation is based solely on what drug causes less drowsiness, I would recommend the new drug, drug A. Treatment A is not significantly different from the placebo (see part (a)) or treatment C (95% CI for mean diff A-C is $(-.14968, 2.15634)$). However, treatment C does appear to cause more drowsiness than treatment A. Boxplots could also be used to show that the median for trtment C is lower than the medians for trtment A & the placebo.

d. In the actual experiment, the drugs were administered on three different days, but any possible day effect was ignored above. Do you think it would be reasonable to conduct a block design using days as blocks in this setting? Briefly explain your answer. (10)

No.

Ideally, we would like to administer all three treatments on the same day (in random order) to eliminate day effects but that might not be physically possible in this study because there are only 24 hours in a day & the treatments cannot be given too close together.

5. Researchers are interested in whether magnetic fields can relieve chronic pain. A sample of patients experiencing post-polio pain syndrome were recruited and randomly assigned to active-magnet and placebo treatments in a double blind experiment. Pain relief was measured on a scale from -10 (a sharp increase in pain) to 10 (pronounced relief). Data are available in P:Data/MATH/STATS/magnet_pain.mtw.



a. Use graphical (a simple sketch will do) and numerical statistical summaries to compare the active-magnet and placebo groups. (15)

	N	mean	s	min	Q1	median	Q3	Max
Active-Magnet	21	6.667	2.288	1	5.5	6	8	10
Placebo	21	1.095	1.578	0	0	0	1.5	5

The boxplot clearly shows that the active magnet scores tend to be higher (more relief) than the placebo group. The mean & median are both higher. There is also more variability in the scores for the AM group. The scores for the AM group are skewed left & the scores for the placebo group are skewed right.

b. Calculate and interpret the 99% confidence intervals for the mean responses of the two groups. What inference can you make concerning the outcome of the experiment? (15)

99% CI for $\mu_{A.M.}$ is (5.24626, 8.08708)

99% CI for $\mu_{placebo}$ is (0.11537, 2.07510)

We are 99% confident that $\mu_{A.M.}$ is in the interval from (5.24626, 8.09). This interval is substantially above the interval ~~that we are confident~~ (.12, 2.08), in which we are 99% confidence that $\mu_{placebo}$ falls. There is a very clear statistically significant difference in the two means.

c. Calculate and interpret the 99% confidence interval for the difference in the means of the two groups. (10)

We are 99% confident that $\mu_{AM} - \mu_{placebo}$ falls in the interval from (3.91953, 7.22352)

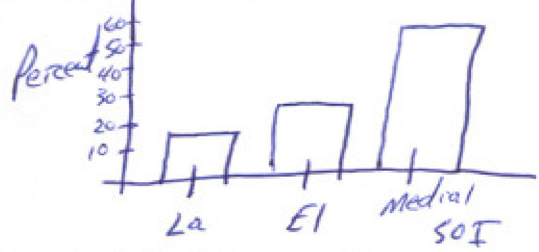
Since zero is not in this interval, we have significant evidence of a difference in the mean scores for the two groups.

6. In the southwestern USA, regional spring precipitation can be predicted up to 9 months in advance from the El Niño Southern Oscillation Index (SOI), which describes shifts in the currents of the southern Pacific Ocean. So-called "El Niño" conditions tend to produce wet springs, while "La Nina" conditions tend to produce dry springs. Because spring weather conditions affect summer food availability, wildlife researchers in northern New Mexico want to know if they can use winter SOI conditions to anticipate encounter rates between humans and black bears in the following summer. This lead-time can enable improved management to facilitate coexistence and reduce mortality associated with the relocation of nuisance bears. For the years1967 – 2001, SOI conditions were classified as El Niño (wet spring predicted), La Niña (dry spring predicted) or Medial (average spring predicted) and the bear-human encounter rate was classified as either High or Low, based on wildlife management and animal control records. Counts of the years are summarized in the following table

	Bear-human encounter rate		
SOI conditions	High	Low	
La Niña	4	2	6
El Niño	2	6	8
Medial	4	16	20
	10	24	34

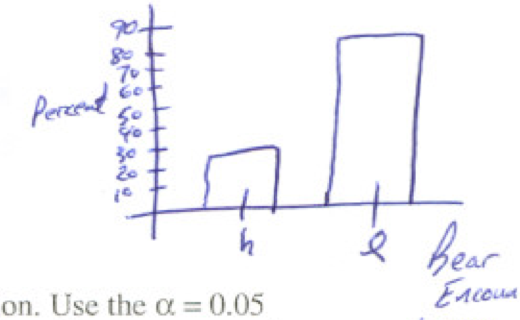
a. Calculate the marginal distribution of SOI conditions for the years 1967 – 2001 and sketch a bar graph of the result. (10)

SOI	La Niña	El Niño	Medial
prob	.1765	.2353	.5882



b. Calculate the conditional distribution of bear-human encounter rates in Medial years and sketch a bar graph of the result. (10)

(Bear | Medial)	high	low
prob	$\frac{4}{20} = .2$	$\frac{16}{20} = .8$



c. Perform the appropriate hypothesis test for this research question. Use the $\alpha = 0.05$ significance level. State your null and alternative hypotheses, describe your test, and summarize and interpret your results. (15)

H_0: no association between SOI & Bear-human encounter

H_1: SOI & Bear-human encounter are associated

$\chi^2 = 4.939$

P-value = .085

Conclusion - We do not have statistically significant evidence of association at the $\alpha = .05$ level even though there are clear differences in the encounter frequencies.

with expected counts ; cells below 5 . Be careful using χ^2 for inference here. Further research would be valuable

$\ast$ $\boxed{\text{One student collapsed table \& used Wilson — Great idea}}$

7. In 1999 a random digit dial phone survey of 1,500 Americans was conducted by People for the American Way to assess knowledge about the theory of evolution and attitudes towards the teaching of evolution and creationism in public schools. Preliminary questions revealed that 720 of the respondents understood what evolution means, while 480 misunderstood the meaning of evolution. Of those that understood what evolution means, 70% were opposed with the decision by the Kansas State Board of Education to remove evolution from the state's science standards. Among those who did not understand the meaning of evolution 55% were opposed to the decision.

$n=1500$
$x=720$
Some students ignored the other 300 & used $n=1200$ without explanation

a. Calculate and interpret the 95% confidence interval for the proportion of Americans that understood what the term evolution means. (10)

We are 95% confident that the proportion of Americans that understood what the term evolution means is in the interval from (.454717, .505283). 95% confident that Between 45% & 51% of Americans understand the term evolution.

b. If the overall survey had a sample size of 1,000 rather than 1,500, would the confidence interval calculated in part (a) be wider or narrower? Justify your response. (10)

$100*C\%$ CI for p is $\hat{p} \pm \underbrace{z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}_{\text{margin of error}}$

If we assume that $\hat{p}$ stays roughly the same & the same Confidence level is used, then decreasing the sample size will increase the margin of error. Thus, the C.I. would be wider.

c. How large would the sample size have to be to attain a 1% margin of error? (10)

Conservative approach:

$$n = \left(\frac{1.96}{.01}\right)^2 * \frac{1}{2} * \frac{1}{2} = \boxed{9604}$$

Other approaches (based on previous $\hat{p}$)

$$n = \left(\frac{1.96}{.01}\right)^2 * .48 * .52 = 9588.6336 \Rightarrow \text{use } n = 9589$$

OR

$$n = \frac{(1.96)^2}{(.01)^2} * .6 * .4 = 9219.84 \Rightarrow \text{use } 9220$$

d. Is the proportion opposed to the Kansas State Board of Education decision significantly higher among those who have a correct understanding of evolution? Perform an appropriate hypothesis test to address this question. State your null and alternative hypotheses, describe your test, and summarize and interpret your results. (15)

$H_0: P_u = P_M$, where P_u = Proportion of those with a correct understanding of evolution who oppose KSBE decision

P_M = proportion of those with a misunderstanding of evolution who oppose KSBE decision.

$H_1: P_u > P_M$.

Test Stat: $n_u = 720$ $X_u = .7(720) = 504$

$n_M = 480$ $X_M = .55(480) = 264$

# of successes & # of failures are both larger than 15.

use pooled estimate for p.

$Z = 5.30$

P-value $= 0.00$.

Since $0.00 < .05$, we reject H_0 and conclude that the proportion opposed to the KSBE decision is significantly higher among those who understand evolution.

8. You are building a brick wall. Standard "3 inch" bricks are 2 5/8 inches high to allow for a 3/8 inch layer of mortar between bricks. The height of a single course of bricks and mortar varies according to a normal distribution with a mean of 3 inches and a standard deviation of 0.05 inch. Heights of successive courses (rows) of bricks and mortar are independent. In the end, your wall is supposed to be 8 feet (96 inches or **32 courses**) high.

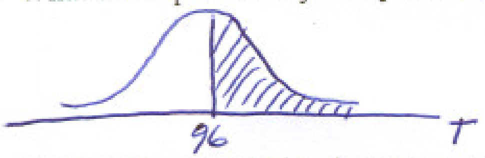
a. What is the probability distribution for the final height of your wall? (10)

Let T = total height of wall $= X_1 + X_2 + \cdots + X_{32}$

CLT says that T will be approximately normal with

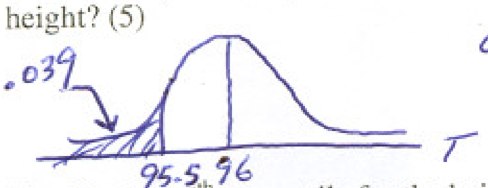
$\mu_T = 32(3) = 96$ inches and $\sigma_T = \sqrt{32}\,(.05) = .2828$ inch

b. What is the probability that your wall is 96 inches high or higher? (5)



$P(T > 96) = P(Z > 0) = .5$

c. What is the probability that your wall is more than half an inch short of the designed height? (5)

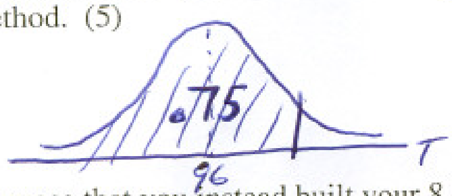


cdf of $N(96, .2828)$

constant 95.5

.0385276

d. Identify the 75th percentile for the height of walls constructed using the 32 course method. (5)



invcdf

96.1907

e. Suppose that you instead built your 8 foot wall using 16 courses of "6 inch" bricks, with the same standard deviation as above. Would your probability of being half an inch short be higher or lower than in the case of the three inch bricks? You do not necessarily need to do the probability calculation, just provide your reasoning. (10)

$T_{16} \approx N\left(\mu_{T_{16}} = 16(6) = 96,\ \sigma_{T_{16}} = \sqrt{16}\times(.05) = .20\right)$

The std deviation decreases which means the dist will be more concentrated about the desired ht of 96 inches. Thus, the probability of being half an inch short will be lower.